Non–Singular Charged Black Hole Solution for Non–Linear Source

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A non–singular exact black hole solution in General Relativity is presented. The source is a non–linear electromagnetic field, which reduces to the Maxwell theory for weak field. The solution corresponds to a charged black hole with $|q| \leq 2s_c m \approx 0.6\, m$, having metric, curvature invariants, and electric field bounded everywhere.

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The existence of singularities is one of the basic failure of the General Relativity Theory [1]; it appears to be an inherent property to most of the solutions of the Einstein equations. The Penrose censorship conjecture establishes that these singularities must be dressed by event horizons; no causal connection exits with the interior of a black hole, and thus the pathologies occurring at the singular region have no influence on the exterior region, and the Physics outside is well–behaved (cf. [2] for a review on the recent status of this conjecture). Nevertheless, the whole space–time has to contain also the interior of black hole, since gravity permits physical objects to fall inside. Hence, we need to know what happens in this falling process. However, the singular behavior of the known black hole solutions made impossible a good description of it, this has been interpreted has a breakdown of General Relativity.

Some regular black hole models has been proposed [3–8] in order to understand this process. All of them have been referred as "Bardeen black holes" [9], since Bardeen was the first author to produce a surprisingly regular black hole model [3]. No one of these models is an exact solution to Einstein equations; there is no known physical sources associated with any of them. The solution to this problem has usually been suggested by finding more general gravity theories avoiding the existence of singularities. The best candidate today to produce singularity–free models, even at the classical level, due to its intrinsic non–locality is string theory [10]. There are examples in other contexts, for instance, in $N = 1$ supergravity domain wall solutions with horizons but no singularities have been found (cg. [11] and references therein), other example is in exact conformal field theory [12].

We show in this letter that there is no need to desist from General Relativity to solve the singularity problem. By assuming an appropriate non–linear source of matter, which in the weak field approximation becomes the usual linear theory, one can achieve a singularity–free black hole solution to Einstein equations coupled with a non–linear electrodynamics. Previous attempts in this direction with non–linear electrodynamics either have totally been unsuccessful or only partially solve the singularity problem [13–15].

We derived our solution using a non–linear electrodynamic source described by the action [16]

$$\mathcal{S} = \int dv \left(\frac{1}{16\pi} R - \frac{1}{4\pi} (2P\mathcal{H}_P - \mathcal{H}) \right), \tag{1}$$

where R is scalar curvature, $P \equiv \frac{1}{4} P_{\mu\nu} P^{\mu\nu}$,

$$\mathcal{H}(P) = \frac{P\, \mathrm{e}^{-s \sqrt[4]{-2q^2 P}}}{\left(1 + \sqrt{-2q^2 P}\right)^{5/2}} \left(1 + \sqrt{-2q^2 P} + \frac{3}{s} \sqrt[4]{-2q^2 P} \right) \tag{2}$$

is a function describing the source, and $\mathcal{H}_P \equiv \partial\mathcal{H}/\partial P$. In (2) $s \equiv |q|/2m$, q and m are free parameters which will be associated with charge and mass respectively. The last function satisfies the plausible condition, needed for a non–linear electromagnetic model, of correspondence to Maxwell theory, i.e., $\mathcal{H} \approx P$ for weak fields ($P \ll 1$). In this description the usual electromagnetic strength is given by $F_{\mu\nu} \equiv \mathcal{H}_P P_{\mu\nu}$.

In order to obtain the desired solution we consider a static and spherically symmetric configuration

$$\boldsymbol{g} = -A(r)\boldsymbol{dt}^2 + A(r)^{-1}\boldsymbol{dr}^2 + r^2\boldsymbol{d\Omega}^2, \tag{3}$$

and the following ansatz for the anti–symmetric field $P_{\mu\nu} = 2\delta^0_{[\mu}\delta^1_{\nu]}D(r)$. With these choices the Einstein–non–linear electrodynamic field equations following from action (1),

$$G_\mu^{\ \nu} = 2(\mathcal{H}_P P_{\mu\lambda}P^{\nu\lambda} - \delta_\mu^{\ \nu}(2P\mathcal{H}_P - \mathcal{H})), \qquad \nabla_\mu P^{\alpha\mu} = 0, \tag{4}$$

are directly integrated, yielding

$$\boldsymbol{g} = -\left(1 - \frac{2mr^2 \mathrm{e}^{-q^2/2mr}}{(r^2+q^2)^{3/2}}\right)\boldsymbol{dt}^2 + \left(1 - \frac{2mr^2 \mathrm{e}^{-q^2/2mr}}{(r^2+q^2)^{3/2}}\right)^{-1}\boldsymbol{dr}^2 + r^2\boldsymbol{d\Omega}^2, \tag{5}$$

$$D = \frac{q}{r^2}. \tag{6}$$

We can note that q actually plays the role of the electric charge; a calculation of the electric field $E = F_{01} = \mathcal{H}_P D$ gives

$$E = \frac{q\,\mathrm{e}^{-q^2/2mr}}{(r^2+q^2)^{7/2}}\left(r^5 + \frac{(60\,m^2 - q^2)r^4}{8\,m} + \frac{q^2 r^3}{2} - \frac{q^4 r^2}{4m} - \frac{q^4 r}{2} - \frac{q^6}{8m}\right), \tag{7}$$

from which two facts follow: the electric field is bounded everywhere, and asymptotically behaves as $E = q/r^2 + O(1/r^3)$, i.e., a Coulomb field with electric charge q. With regard to the metric, it can be noted that it asymptotically behaves as the Reissner–Nordström solution, i.e., $g_{00} = 1 - 2m/r + q^2/r^2 + O(1/r^3)$, so the parameters m and q can be correctly associated with mass and charge respectively.

We will show that for a certain range of mass and charge our solution is a black hole, which moreover is non–singular everywhere. Making the substitution $x = r/|q|$, $s = |q|/2m$ we write

$$-\,g_{00} = A(x,s) \equiv 1 - \frac{1}{s}\frac{x^2 \mathrm{e}^{-s/x}}{(1+x^2)^{3/2}}. \tag{8}$$

For any value of s, the last function has a single minimum for $x_{\mathrm{m}}(s) = (s + (s^2+6)/R(s) + R(s))/3$, where $R(s) \equiv (s^3 + (45/2)s + 3\sqrt{3(s^4 + (59/4)s - 8)})^{1/3}$. For $s < s_{\mathrm{c}}$ this minimum is negative, for $s = s_{\mathrm{c}}$ the minimum vanishes and for $s > s_{\mathrm{c}}$ the minimum is positive, where $s_{\mathrm{c}} \approx 0.3$ is the solution to the equation $A(x_{\mathrm{m}}(s), s) = 0$. Calculating the curvature invariants R, $R_{\mu\nu}R^{\mu\nu}$, and $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$ for metric (5) one establishes that all of them are bounded everywhere; thus for $s \le s_{\mathrm{c}}$ the singularities appearing in (5) (the vanishing of A) are only coordinates–singularities describing the existence of horizons, and we are in presence of black hole solutions for $|q| \le 2s_{\mathrm{c}}m \approx 0.6\,m$. For these values of mass and charge we have, under the strict inequality $|q| < 2s_{\mathrm{c}}m$, inner and event horizons for the Killing field $\boldsymbol{k} = \boldsymbol{\partial/\partial t}$, defined by $-k_\mu k^\mu = A(r) = 0$. For the equality $|q| = 2s_{\mathrm{c}}m$, they shrink into a single horizon, where also $\nabla_\nu(k_\mu k^\mu) = 0$, i.e., this case corresponds to an extreme black hole as in the Reissner–Nordström solution. The extension of the metric beyond the horizons, up to $r = 0$, becomes apparent by passing to the standard advanced and retarded Eddington–Finkelstein coordinates, in terms of which the metric is well–behaved everywhere, even in the extreme case. The maximal extension of this metric can be achieved by following the main lines presented in [17] for the Reissner–Nordström solution, taking care, of course, of the more involved integration of the tortoise coordinate $r^* \equiv \int A^{-1} dr$ in our case. Summarizing, our space–time possesses the same global structure as the Reissner–Nordström black hole except that the singularity, at $r = 0$, of this last solution has been smoothed out and $r = 0$ is now simply the origin of the spherical coordinates. This kind of metrics is not new (cf. [9] for a review) but the new feature in this case is that it is an *exact solution*, as opposed to the previous ones that are only non–singular black hole *models*.

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